Explanation of online resource with chapter 7+8 of
A Spiky World: An Introduction to Urban and Geographical Economics

Follow these steps

1. Open the accompanying excel file *coremodel.xlsm*.
2. You may be prompted by a notification to accept the use of macros: **do this**.
(Otherwise, the built-in macros such as *"solve short term (for given λ_1)"* and *"solve long term (for all λ_1)"* will not work)
3. The excel file uses the built-in **solver add-in** of Microsoft Excel. You need to **load the solver add-in in Excel** first. No additional downloads are required for this: it is a built-in feature of excel software. You need to tick a box in the options menu to be able to use it. Instructions on how to do this are available here.
(full hyperlink: https://support.office.com/en-us/article/load-the-solver-add-in-in-excel-612926fc-d53b-46b4-872c-e24772f078ca)

(1) Core model, two regions, short-term, with congestion

Recall that the normalized short-run equilibrium is characterized by equations (8.23 – 8.25):

$$Y_r = \delta\lambda_r W_r + (1-\delta)\phi_r \tag{8.23}$$

$$I_r = \left[\sum_{s=1}^{R} \lambda_s^{1-\tau\varepsilon} T_{rs}^{1-\varepsilon} W_s^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{8.24}$$

$$W_r = \lambda_r^{-\tau}\left[\sum_{s=1}^{R} Y_s T_{rs}^{1-\varepsilon} I_s^{\varepsilon-1}\right]^{\frac{1}{\varepsilon}} \tag{8.25}$$

This set of equations for each region $r = 1, \ldots, R$ determines income level Y_r, price index I_r, and wage rate W_r. The parameters that determine this are the share of income spent on manufactured goods (δ), the share of total manufacturing labor force (λ), the share of total food labor force (ϕ), the iceberg transport costs to go from region to region (T_{rs}: note that $T_{rs} = T^{D_{rs}}$ and D_{rs} is the economic distance between region r and s, with $D_{rs} = 1$ for two regions), the congestion parameter τ which lies between 0 and 1 where higher values indicate stronger effects of congestion, and finally the elasticity of substitution or price elasticity of demand (ε), which is equal to $\frac{1}{1-\rho}$ where ρ is the substitution parameter with $0 < \rho < 1$.

To find solutions for Y_r, I_r and W_r, we input values for the parameters and solve the system. These are six equations in six unknowns, and almost impossible to solve analytically. Hence, we turn to computer software – in this case Excel – to numerically solve this system for us. This is handy, because it also allows us to look at model dynamics: we can simply change the input parameters and see how the outcomes respond.

4. Make sure you are on the tab "two regions" in the accompanying excel file.
5. Click "solve short term" once.

This sheet can be divided into five parts: the **input parameters**, the **model solver**, the solutions of the model **for a full range of manufacturing labor shares**, **graphs belonging to this full solution**, and **tomahawk diagrams**.

The solver applies an iterative seeking procedure to find a solution for the 6 unknowns in our system of 6 equations. Outcomes for this short term are displayed in the G-column, titled **Solution**. To illustrate how the solver works, let us take the base scenario parameter configuration also used in chapter 7 of the book. These parameters are: $\delta = 0.4$, $\varepsilon = 5$, $T = 1.7$, $\phi_1 = \phi_2 = 0.5$, and $\tau = 0$.

Solve short term solves the model for the value you plug in for λ_1. This makes sense: λ_1 stands for the share of total manufacturing labor force in region 1. In the short term, this share is fixed, as workers in manufacturing cannot move across regions in the short term. Let us take $\lambda_1 = 0.500$. This means that $\lambda_2 = 0.500$ as well. The regions are completely symmetrical, as all parameters are equal across regions. Hence, the solution should be completely symmetrical: $Y_1 = Y_2, I_1 = I_2, W_1 = W_2, \frac{w_1}{w_2} = 1$.

Results of solving the model using this parameter configuration are given in figure 2 below. You can confirm that you get the same outcome using your own excel file by clicking *Solve short term*.

Technical summary – the iterative seeking procedure (not important, unless you are interested)

Writing equations (8.23-8.25) in residual form yields:

$$Y_1 - [\delta\lambda_1 W_1 + (1-\delta)\phi_1] = 0$$
$$Y_2 - [\delta\lambda_2 W_2 + (1-\delta)\phi_2] = 0$$
$$I_1 - [\lambda_1^{1-\tau\varepsilon} W_1^{1-\varepsilon} + \lambda_2^{1-\tau\varepsilon} T_{12}^{1-\varepsilon} W_2^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} = 0$$
$$I_2 - [\lambda_2^{1-\tau\varepsilon} W_2^{1-\varepsilon} + \lambda_1^{1-\tau\varepsilon} T_{21}^{1-\varepsilon} W_1^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} = 0$$
$$W_1 - \lambda_1^{-\tau}[Y_1 I_1^{\varepsilon-1} + Y_2 T_{12}^{1-\varepsilon} I_2^{\varepsilon-1}]^{\frac{1}{\varepsilon}} = 0$$
$$W_2 - \lambda_2^{-\tau}[Y_2 I_2^{\varepsilon-1} + Y_1 T_{21}^{1-\varepsilon} I_1^{\varepsilon-1}]^{\frac{1}{\varepsilon}} = 0$$

By definition, the model is solved if the residuals of all individual equations are indeed equal to zero- that is, using the parameter inputs and the values for Y_r, I_r and W_r to calculate the left sides of the above equations, the right sides should all be zero. These residuals are generated under *"Outcomes of equations (8.23-8.25) in residual form"*.

The solution engine used is the *GRG Nonlinear engine*. GRG stands for generalized reduced gradient algorithm[1]. This method looks at the gradient of the objective function subject to a set of constraints (in our case, the six constraints that the individual residuals are smaller than 0.0001) as the variables' values change. It determines that an optimum is reached when the partial derivatives equal zero. As objective function we use the sum of all individual residuals, which must be equal to zero at the optimum.

Stopping criterion
A solution is found when all constraints and optimality conditions are satisfied. More specifically, a solution is found when the relative change in the objective function is less than 0.0001 in the last 5 iterations. Note that because this is a numerical solution method, solutions are not exactly equal to each other and residuals are not exactly equal to zero- this is a tradeoff between computational speed and preciseness. A final question is: is this the global optimal solution? It turns out that giving values that fall within the *"parameter value domain"* as defined in column C (note: these domains correspond with economic intuition) always leads to at most 1 optimum found by this solution method, which is also the globally optimal solution.

[1] - Lasdon, L. S., Waren, A. D., Jain, A., & Ratner, M. (1976). *Design and testing of a generalized reduced gradient code for nonlinear programming* (No. SOL-76-3). Stanford University of California Systems Optimization Lab

Figure 1 – Excel solver of core model of geographical economics, two regions, including congestion

The model solver				
Variables	**Solution**		**Outcomes of equations (7.2 - 7.4) in residual form**	
Y_1	0.50003		1E-05	
Y_2	0.49996		-5E-05	
I_1	1.15617		1E-05	
I_2	1.15613		1E-05	
W_1	1.00010		1E-05	
W_2	1.00005		1E-05	
		Sum of residuals	-4.5E-09	
w_1/w_2	1.00003			

Figure 2 – Solve single case, with $\delta = 0.4$, $\varepsilon = 5$, $T = 1.7$, $\lambda_1 = \lambda_2 = 0.5$, $\phi_1 = \phi_2 = 0.5$, and $\tau = 0$.

Indeed, we find that $Y_1 = Y_2, I_1 = I_2, W_1 = W_2$ and $\frac{w_1}{w_2} = 1$. Results are presented in figure 2 above. Figure 1 above explains more carefully the solution algorithm and the columns in the **model solver**.

To find out the effects of changing other input parameters than λ, we can take a value of $\lambda_1 \neq 0.500$. Say, for example, we take $\lambda_1 = 0.250$. That is, the share of total manufacturing workers located in region 1 is 0.25; then the share of total manufacturing workers located in region 2 is 0.75. Clicking *Solve short term* shows us that this leads to $Y_2 > Y_1$. Nominal income in region 2 is higher than in region 1. On the other hand, nominal wages and the exact price index of manufactures are higher in region 1 than in region 2. Of course, a multitude of questions is possible by changing one or more input parameters. You can use the model solver to quickly look at any of these situations.

When comparing cases, it is helpful to copy outcomes (and their corresponding parameter configurations) to some empty cells, as each time you solve the model, the old outcomes in the Solution column get replaced. Indeed, this will prove helpful in answering the following questions below.

Exercises

(1a)
Use the case where $\lambda_1 = 0.25$ (therefore, $\lambda_2 = 0.75$). Leave the other parameters at their initial values ($\delta = 0.4, \varepsilon = 5, T = 1.7, \phi_1 = \phi_2 = 0.5$).
Calculate outcomes for Y_r, W_r, I_r and w_1/w_2. How does this short run equilibrium change if we introduce congestion? Take $\tau = 0.1$ to test this.

(1b)
Again use the case where $\lambda_1 = 0.25$, and the other parameter values are at their initial values ($\delta = 0.4, \varepsilon = 5, T = 1.7, \phi_1 = \phi_2 = 0.5, \tau = 0$).
How does this short run equilibrium change if varieties of goods become more complementary?

(1c)
Again use the case where $\lambda_1 = 0.25$, and the other parameter values are at their starting level ($\delta = 0.4, \varepsilon = 5, T = 1.7, \phi_1 = \phi_2 = 0.5, \tau = 0$).
Reason intuitively what will happen to nominal income, nominal wages and the exact price indices in both regions if the government starts collecting toll on the road network link between the two regions. Assume toll does not cause congestion. Confirm your reasoning with the model solver.

Answers are given at the end of the document

(2) Two regions, long-term

Much like the process described in section 7.7, the macro titled **Solve long term** numerically solves the model for 30 different values of λ_1 between 0.001 and 0.999. Note that it does not matter here what value you use as input for λ_1 as the model is solved for 30 prespecified values of λ_1. Outcome values are saved in the **For the full range of manufacturing labor shares** block. Below that, **graphs for the full range of manufacturing labor force shares** are automatically plotted.

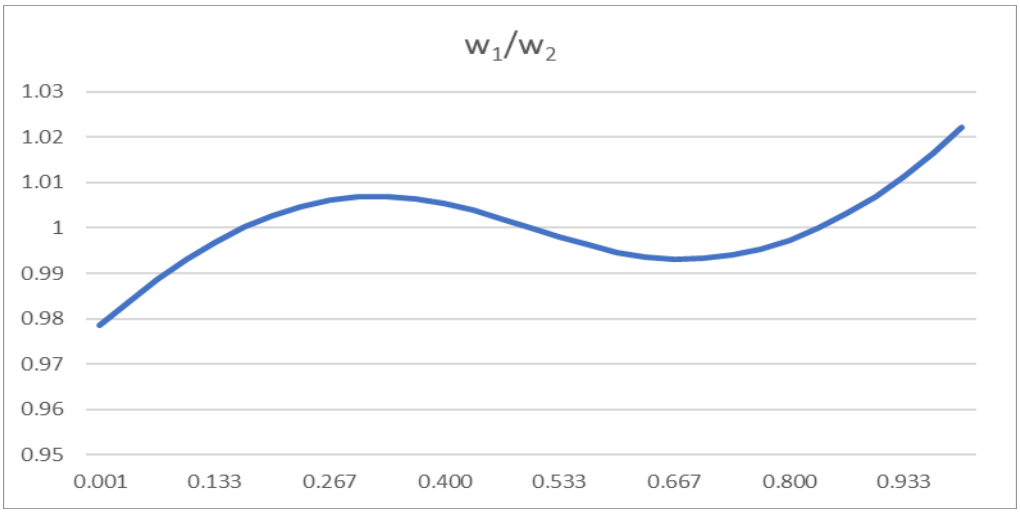


Figure 3 – Relative real wage in region 1 (w_1/w_2) vs. share of manufacturing workers in region 1 (λ_1)

Indeed, for the input parameters we used before ($\delta = 0.4$, $\varepsilon = 5$, $T = 1.7$, $\phi_1 = \phi_2 = 0.5$, $\tau = 0$), we find the exact same graph as in figure 7.6 of the book: see figure 3 above. What can we learn from this figure, which is essentially a combination of short-term equilibria? In the model, the long-run component is that manufacturing workers are mobile across regions. Specifically, when relative real wages are higher in region 1 ($w_1/w_2 > 1$), manufacturing workers move to region 1: the equilibrium moves to the *right* along the graph. When relative real wages are higher in region 2 ($w_1/w_2 < 1$), the equilibrium moves to the *left* along the graph.

Hence, we find a **long-run equilibrium** if the real wage for the mobile workforce is the same in all regions *where there is a mobile workforce*. Or, in other words, there is (1) a long-run equilibrium for relative real wages (w_1/w_2) equal to one, or (2) for complete agglomeration of manufacturing workers ($\lambda_1 = 0$ or $\lambda_2 = 0$). Only some of these equilibria are stable. The equilibrium at $\lambda_1 = 0$ is stable: relative real wages are smaller than one, thus real wages are higher in region 2, thus manufacturing workers will want to stay in region 2. Using similar reasoning, the equilibrium at $\lambda_1 = 1$ is also stable. The equilibrium at $\lambda_1 \approx 0.167$ is unstable: moving towards region 1 leads to higher relative real wages in region 1 and moving towards region 2 leads to higher relative real wages in region 2. Hence, if one worker moves to another region, many others will want to suddenly follow: the equilibrium is unstable. Using similar reasoning, the equilibrium at $\lambda_1 \approx 0.833$ is unstable. Finally, the symmetric equilibrium at $\lambda = 0.500$ is stable: moving towards region 1 leads to higher relative real wages in region 2, so one would want to move back, while moving towards region 2 leads to higher relative real wages in region 1, in which case one would also want to move back.

Thus, we can now find out what the stable long-run equilibrium will be given any initial allocation of manufacturing workers over space. In the above example, for $\lambda < 0.167$, the long-run equilibrium will be at full agglomeration of manufacturing in region 2 ($\lambda_1 = 0$). For λ_1 between 0.167 and 0.833, the long-run equilibrium will be the symmetric one at $\lambda_1 = 0.500$. For $\lambda_1 > 0.833$, the long-run equilibrium will be full agglomeration in region 1 ($\lambda_1 = 1$).

Technical summary: errors at $\lambda_r = 0$ or 1

Because excel returns an error for the calculation 0^0 (which, for $\tau = 0$, forms the start of expression (8.25)), we cannot find the exact values for Y_r, I_r and W_r associated with the full agglomeration equilibria for the case where $\tau = 0$; however, the values for $\lambda_1 = 0.001$ and $\lambda_1 = 0.999$ give a good approximation. This is why the values for λ in the "*For the full range of manufacturing shares"* box start at 0.001 and end at 0.999 instead of starting at 0 and ending at 1.

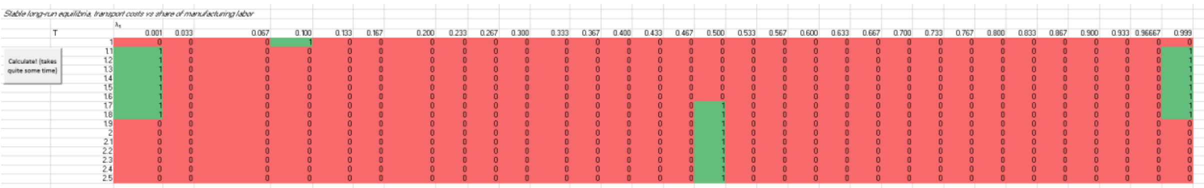
The solver also gives an output for whether one of the 30 chosen values of λ_1 is a stable long-run equilibrium or not. This is shown in row 28, titled **Stable long run equilibrium?**. It takes a value of 1 for stable long-run equilibria and 0 otherwise.

Technical summary: stable long-run equilibrium?

The expression we use to identify whether there is a stable long-run equilibrium follows the reasoning we used above in interpreting figure 3 of this document. Specifically, a stable long run equilibrium is a λ_1 for which w_1/w_2 crosses the $w_1/w_2 = 1$ line with negative slope compared to the previous and subsequent value of λ_1. As we use 30 chosen values for λ_1, this will give the long-run equilibrium values for λ_1 with a precision of at least 0.033, or 3.3% of the total manufacturing labor share. Assuming the w_1/w_2 function is locally linear (which is generally the case), the precision increases to at least 0.0167, or 1.67% of the total manufacturing labor share. In other words, this procedure will find long-run stable equilibrium values of λ_1 within 0.0167 labor share of the true long-run stable equilibrium. Another stable long-run equilibrium is $\lambda_1 = 0.001$ (actually $\lambda_1 = 0$) if the slope there is positive and the value below 1, or $\lambda_1 = 0.999$ (actually $\lambda_1 = 1$) if the slope there is positive and the value above 1.

6. Click "solve long term" once. You will see the relative wage graph appearing, and you can replicate figure 3 of this document if you follow the associated parameter configuration.
7 Scroll down in the excel file. You will find a part titled **Stable long-run equilibria, transport costs vs share of manufacturing labor**. Click "Calculate", noting that this takes excel some time to complete!

The table for **Stable long-run equilibria, transport costs vs share of manufacturing labor** presents the stable long-run equilibria calculated for the range of 30 values of λ_1 for a range of 16 different values for transport costs. Because this requires sixteen times thirty times a completion of the solution algorithm, this procedure takes a few minutes. Stable long-run equilibria for the given values of λ_1 and T receive a value of 1 and a green background; other cells receive value of 0. From this, we can see the Tomahawk diagram of figure 7-9 of the book, as shown in figure 4 below.


Stable long-run equilibria, transport costs vs share of manufacturing labor

Calculate! (takes quite some time)

T \ λ_1	0.001	0.033	0.067	0.100	0.133	0.167	0.200	0.233	0.267	0.300	0.333	0.367	0.400	0.433	0.467	0.500	0.533	0.567	0.600	0.633	0.667	0.700	0.733	0.767	0.800	0.833	0.867	0.900	0.933	0.96667	0.999
1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
1.2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
1.3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
1.4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
1.5	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
1.6	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
1.7	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
1.8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
1.9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2.2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2.3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 4 – The Tomahawk diagram, transport costs vs share of manufacturing labor

Using this Tomahawk diagram, we can see how the distribution of stable long-term equilibria over the range of manufacturing shares change as other input parameters (in this case Transport costs) change. Scrolling further down, you will find further tomahawk 'diagrams' for a range of δ, ε, ϕ_r, and τ.

Technical summary: missing values in my Tomahawk diagram?

One may encounter errors in the tomahawk diagrams for some parameter inputs, especially close to corner solutions ($\lambda_1 = 0$ or $\lambda_1 = 1$). This is because of computational limitations imposed by excel software and the excel solver. Specifically, in the functions based on equations (8.23-8.25), values for some of terms calculated in intermediate steps may become so close to zero that Excel does not differentiate them from zero anymore, which may lead to division by zero or 0^0, both creating an error.

Exercises

(2a)
Take the input parameters we used before ($\delta = 0.4$, $\varepsilon = 5$, $\phi_1 = \phi_2 = 0.5, \tau = 0$), but increase the transport costs to 1.9. What happens to the long-run equilibria when compared to Figure 3 above (or Figure 7.6 of the book), which used $T = 1.7$? Why do you think this happens?

(2b)
Take the input parameters we used before ($\delta = 0.4$, $\varepsilon = 5$, $T = 1.7$, $\phi_1 = \phi_2 = 0.5$), but now introduce congestion ($\tau = 0.01$). Construct the figure of the relative wage rate again. What are the stable long-run equilibria?

(2c)
Take the input parameters we used before ($\delta = 0.4$, $\varepsilon = 5$, $\phi_1 = \phi_2 = 0.5$), but now introduce congestion ($\tau = 0.1$). Construct the Tomahawk diagram of stable long-run equilibria for a range of transport costs T again. Compare your results to the results in figure 4 of this document. What changed? Can you give an intuitive explanation for these changes, knowing that congestion caused them?

Answers are given at the end of the document

Of course, you are free to play around with any parameter specification you like. For example, one can recreate any of the figures from the book relating to the two-city core model. The underlying idea is that you get a feel for model dynamics: how the spatial equilibria change if conditions (in terms of transport costs, congestion, elasticity of substitution between goods, etcetera) are different.

Indeed, if we extend this model to many cities, we might be able to explain the real-world locational distribution of cities and their population sizes using this model. Hence, a next step is to analytically estimate the outcome of the model for more than two cities. Unfortunately, excel software is not very appropriate for this. Although the excel file does contains an attempt at creating a solvable four-region model, limitations of excel software mentioned earlier in the green boxes, specifically those on handling numbers very close to zero, are amplified in the four-region model. Hence, using the excel solver algorithm for four regions almost always fails to find to a correct solution, especially for the long term. It is advised to use more advanced software instead. For example, one can code the core model for many regions in MATLAB, Python, etcetera. Combining this more advanced software with GIS software can then be used to assess real-world examples, as shown in chapter 9 of the book.

(3) Four regions

(Note: this part is optional material, it does not function well and only serves to show that using other software is advisable for multi-region modelling)

Navigate to the excel tab "Four regions". This tab contains the same model as before, but for four instead of two regions. This increases the number of equations created by (8.23-8.25) to twelve; still 3 for each region. An illustration of these four regions is given in figure 5 below. The distance between each region is normalized to one. Note that this is a simplified case of the *pancake economy* example in the book, figure 11.4. Again, **Solve short term** finds the short-run equilibrium for given inputs of λ_r.

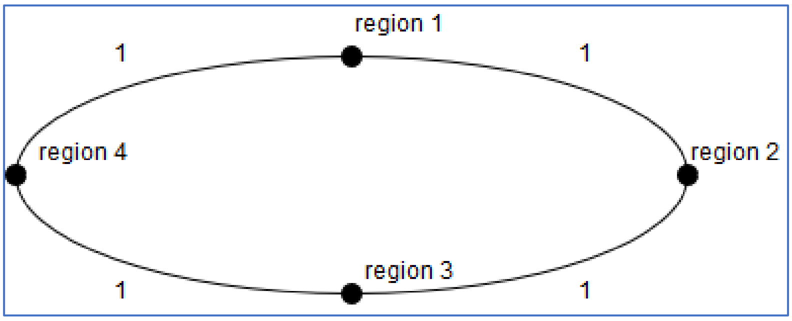


Figure 5 – The pancake economy, four regions

As example, let us choose the completely symmetric case of equal labor shares both in manufacturing and in food. Note that distances are also input here: following figure 5 above, the distance between region 1 and 2 is one, the distance between region 1 and 3 is two, and the distance between region 1 and 4 is one. Again, one can play around with different parameter specifications to see what happens to the outcomes of the model. Interpreting where the long-term stable equilibria lie is much harder in the four-region case. Namely, workers in the manufacturing sector have four instead of two regions to choose from. Excel is only able to provide an algorithm such as the one we used in the two-region case, which often does not lead to an equilibrium because the process does not converge, or because the process suffers from small number issues.

Exercise

(3)
A transport link of distance 1 is constructed between regions 1 and 3, as indicated in figure 6 below. Assume that the rest of the input parameters is still completely symmetric ($\delta = 0.4, \varepsilon = 5, \tau = 0, \phi_r = 0.25, \lambda_r = 0.25, T = 1.7$). What happens to the relative wage rate in regions 1 and 3 with respect to regions 2 and 4? How does lowering transport costs or setting a higher initial manufacturing labor force λ_1 affect this effect?

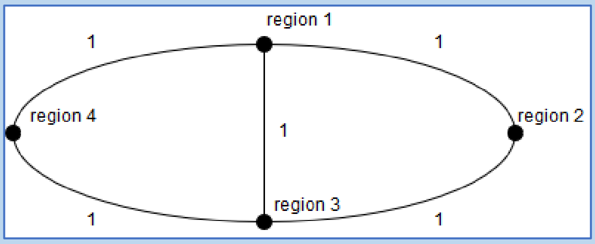


Figure 6 – The pancake economy, four regions, with new link

Answers are given at the end of the document

Answers to questions

Exercises

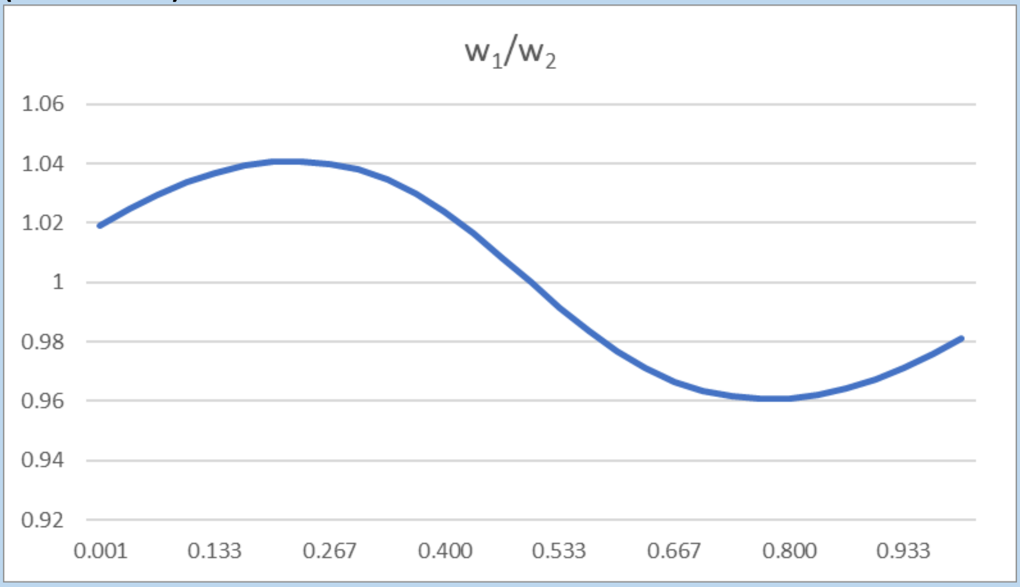
(1a) The exact price indices of manufactures both decrease ($I_1 = 1.26, I_2 = 0.97$), nominal income levels converge slightly ($Y_1 = 0.4175, Y_2 = 0.5825$), and nominal wages diverge ($W_1 = 1.18, W_2 = 0.94$). Real wages are relatively much higher ($\frac{W_1}{W_2} = 1.123$) in region 1. Apparently, introducing congestion leads to higher real wages in the region with the lower share of total manufacturing workers. This makes sense, as congestion leads to welfare losses as cities become more crowded, which in this model is related to the number of manufacturing workers through the total number of manufacturing firms.

(1b) If varieties of goods become more complementary, the substitution parameter ρ becomes lower – you are less likely to substitute varieties of goods since they complement each other more – hence the elasticity of substitution becomes higher. Therefore, we increase ε. Increasing ε and solving the short-term model yields higher relative real wages in the region with less manufacturing workers (region 1): nominal wage levels diverge compared to the initial situation with less complementary goods, national income levels converge, and exact price indices diverge.

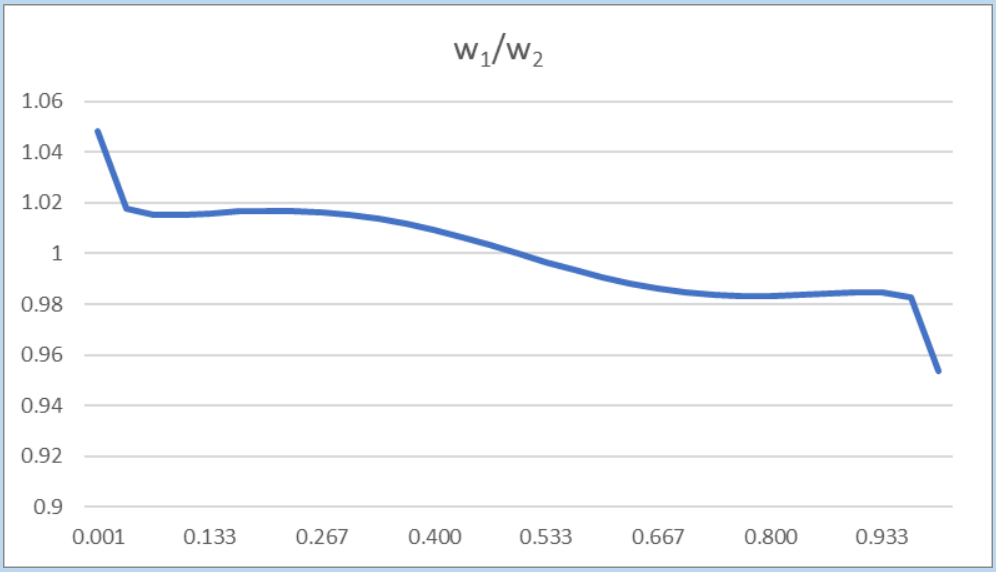
(1c) Collecting toll increases transport costs T. Recall that these transport costs only count for manufactured goods, as farm goods are transported between regions without any cost. Hence, higher T will make it more costly to transport manufacturing varieties across regions. As a result, the price that manufacturing firms charge in the distant region suffers from a higher mark-up. Then, consumers in region 1 spend less on products from region 2 and consumers in region 2 spend less on products from region 1, but since there are more manufacturing products being produced in region 2, nominal income levels become lower in region 2 and higher in region 1: nominal income levels converge. As the price index of manufactures I is essentially a weighted average of the price of locally produced goods and imported goods from other regions, the exact price indices of manufactures as well as nominal wage levels will diverge. The question what happens to relative real wages is more difficult as it depends which effects dominate. It turns out that relative real wages in region 1, the region with less manufacturing workers, decrease- the model solver confirms this.

(2a) We can optically deduce the stable long-run equilibria from the w_1/w_2 graph. Results are presented in the figure below. The long-run equilibria are again at full agglomeration ($\lambda_1 = 0$ and $\lambda_1 = 1$), and at $\lambda_1 = 0.5$ the graph intersects the $w_1/w_2 = 1$ line. However, at $\lambda_1 = 0$, relative real wages are higher in region 1, so manufacturing workers will move to region 1. At $\lambda_1 = 1$, relative real wages are higher in region 2, so manufacturing workers will move to region 2. Indeed, the only stable equilibrium is the symmetric equilibrium at $\lambda_1 = 0.500$. Intuitively, this makes sense: if transport costs are too high, manufacturing goods need to be produced locally. Similar dynamics are found in figure 7.7 of the book, where taking $T = 2.1$ also leads to only one stable equilibrium at spreading ($\lambda_1 = 0.500$).

(2a continued)



(2b) We see here how dependent the relative wages and corresponding (unstable and stable) long-run equilibria are to slight parameter changes by playing around with them. Again, this leads to only one stable long-run equilibrium at the symmetric (spreading) case of $\lambda_1 = 0.500$. Results are presented in the figure below. Note that this is an exact copy of figure 8.10 (panel in top row, middle column).



(2c) Results are presented in the figure below. Note that now, there is only the symmetric spreading equilibrium: apparently, congestion is so heavy that it would not be optimal to have agglomeration of manufacturing in one location even with low transport costs. This makes sense: congestion is created by the number of manufacturing firms and then depresses wage and income in that region. To optimally deal with these negative externalities it would be better here to split the manufacturing firms over space.

(2c continued)

Stable long-run equilibria, transport costs vs share of manufacturing labor

Calculate! (takes quite some time)

T \ λ_1	0.001	0.033	0.067	0.100	0.133	0.167	0.200	0.233	0.267	0.300	0.333	0.367	0.400	0.433	0.467
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2.2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2.3	#GETAL!	######	#GETAL!	0	0	0	0	0	0	0	0	0	0	0	0
2.4	#GETAL!	######	#GETAL!	0	0	0	0	0	0	0	0	0	0	0	0
2.5	#GETAL!	######	#GETAL!	0	0	0	0	0	0	0	0	0	0	0	0

T \ λ_1	0.500	0.533	0.567	0.600	0.633	0.667	0.700	0.733	0.767	0.800	0.833	0.867	0.900	0.933	0.96667	0.999
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.5	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.6	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.7	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.9	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2.1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2.2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2.3	1	0	0	0	0	0	0	0	0	0	0	0	0	######	######	######
2.4	1	0	0	0	0	0	0	0	0	0	0	0	0	######	######	######
2.5	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

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(3) The completely symmetric equilibrium (for inputs $\delta = 0.4, \varepsilon = 5, \tau = 0, \phi_r = 0.25, \lambda_r = 0.25, T = 1.7$) yields relative real wage ratios of 1. As can be confirmed in excel by changing the parameter value for D_{13} to 1, building a new transport link leads to relatively high wage rates in regions 1 and 3 with respect to regions 2 and 4. Introducing lower transport costs T or setting a higher manufacturing labor force λ_1 or λ_3 strengthens this effect.